

Exercise 29

Evaluate the integral.

$$\int_1^4 \frac{2+x^2}{\sqrt{x}} dx$$

Solution

According to part 2 of the fundamental theorem of calculus,

$$\int_a^b f(x) dx = F(b) - F(a),$$

where F is an antiderivative of f . Use the power rule in reverse here: Bump up the exponent by 1 and divide by that exponent.

$$\begin{aligned} \int_1^4 \frac{2+x^2}{\sqrt{x}} dx &= \int_1^4 \left(\frac{2}{\sqrt{x}} + \frac{x^2}{\sqrt{x}} \right) dx \\ &= \int_1^4 (2x^{-1/2} + x^{3/2}) dx \\ &= \int_1^4 2x^{-1/2} dx + \int_1^4 x^{3/2} dx \\ &= \left(\frac{2x^{1/2}}{\frac{1}{2}} \right) \Big|_1^4 + \left(\frac{x^{5/2}}{\frac{5}{2}} \right) \Big|_1^4 \\ &= 4(x^{1/2}) \Big|_1^4 + \frac{2}{5}(x^{5/2}) \Big|_1^4 \\ &= 4(4^{1/2} - 1^{1/2}) + \frac{2}{5}(4^{5/2} - 1^{5/2}) \\ &= 4(1) + \frac{2}{5}(31) \\ &= \frac{82}{5} \end{aligned}$$